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2. Electrical circuits

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2.4 Methods of solving AC electrical circuits

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2.1 Methods of solving DC electrical circuits

2.1.1 Step-by-step simplification

The method is convenient for simpler circuits with single source. The decision procedure is in the circuit replacement by one resistor and calculation of its resistance, the calculation of the total current and solution of specified voltages or currents. The resistors connection is often combined, when some of them can be connected into series, parallel or in delta connection (G. Rizzoni: Fundamentals of electrical engineering).

2.1.2 The solution using Kirchhoff´s laws

Using both Kirchhoff´s laws more complex circuits with more sources can be solved. A disadvantage of this method is more equations needed for the circuit solution. (https://en.wikipedia.org/wiki/Kirchhoff%27s_circuit_laws).

2.1.3 The mesh current method

This method is based on defining image mesh currents for each independent mesh, their calculation by applying Kirchhoff voltage law for the each mesh and after the real currents are solved (G. Rizzoni: Fundamentals of electrical engineering). This method gives a reduction of the number of equations.

2.1.4 The node voltage method

This method is based on a selection of a reference node (usually grounded), next node voltages are defined and by applying Kirchhoff current law for each node the currents are solved (G. Rizzoni: Fundamentals of electrical engineering).

2.1.5 The principle of superposition

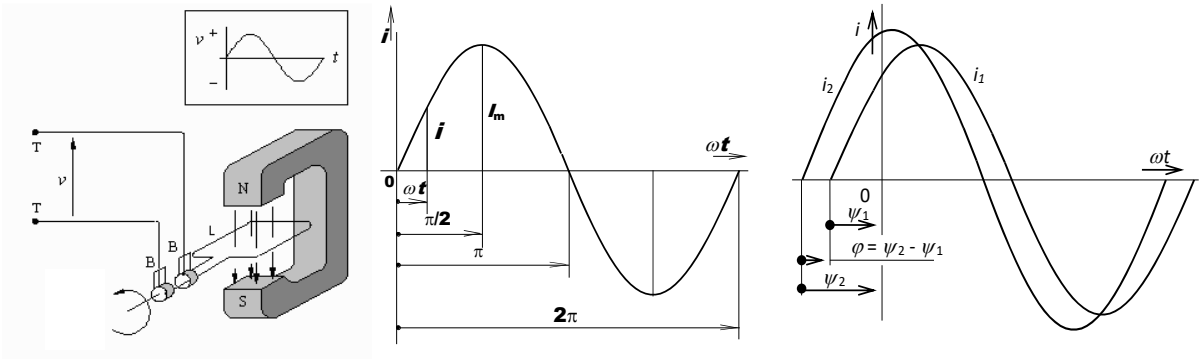
This principle can be used for linear circuits with more sources. The effect of all sources is equal to the sum of effects of separate sources. That means the branch voltage or current is the algebraic sum of the voltages or currents due to each source acting alone. (G. Rizzoni: Fundamentals of electrical engineering).

2.1.6 Thévenin and Norton equivalent circuits

In the case when the current is found only in one branch (load) of a complex circuit, the concept of the equivalent circuit can be used. The Thévenin theorem says that when viewed from the load terminals any circuit composed of ideal sources and resistors can be replaced by an equivalent circuit consisting of an ideal voltage source in series with an equivalent resistance. The equivalent circuit can be also composed of an ideal current source in parallel with an equivalent resistance (The Norton theorem) (G. Rizzoni: Fundamentals of electrical engineering).

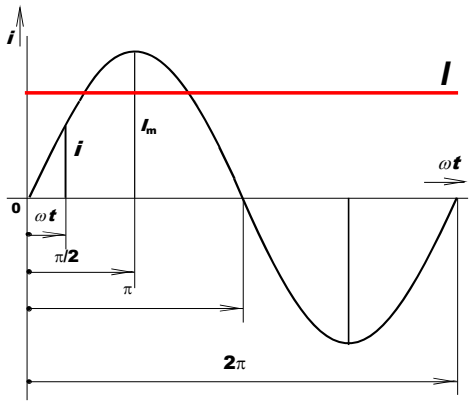
2.2 Alternating circuits: basic definitions and quantities

The alternating circuits excited by the sinusoidal (harmonic) voltage are presented here. A simplified source of the harmonic voltage and waveforms of harmonic currents are shown in the figures.



The sinusoidal current (or voltage of course) is defined as:

$i = I_{\mathrm{m}} \cdot \sin(\omega t)$ $\omega = 2\pi \cdot f = \dfrac{2\pi}{T}$, where i is the instantaneous value, I_{m} is peak value or amplitude, ω is the radian frequency, f is the frequency of current (voltage) equal to 50 Hz in power networks, T is the period. The phase shift is φ. In practice the root-mean-square (rms) value is defined to replace the instantaneous value by the image DC value by the formula:



$$W = R \cdot I^2 \cdot T = \int_0^T R \cdot i^2 \cdot \mathrm{d}\, t$$

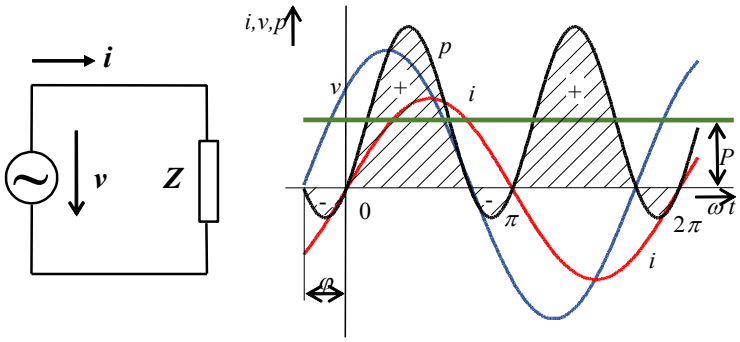
That means the rms value of the current i(t) is the DC current I that causes the same average power (or energy) to be dissipated by the resistor. The relation between the rms value and the peak value is given by the formula:

$$I = \sqrt{\frac{1}{T} \cdot \int_0^T i^2 \cdot \mathrm{d}\, t} = \sqrt{\frac{1}{T} \cdot \int_0^T {I_m}^2 \cdot \sin^2 \omega t \, \mathrm{d}\, t}$$

$$I = \frac{I_{\mathrm{m}}}{\sqrt{2}}$$

2.3 Types of power of AC circuit and their relevance

The instantaneous power is the product of the instantaneous voltage and current in a load given by the formula:



$v = V_{\mathrm{m}} \cdot \sin(\omega t + \varphi)$, $i = I_{\mathrm{m}} \cdot \sin \omega t$, $p = v \cdot i \rightarrow p = V \cdot I \cdot \cos\varphi - V \cdot I \cdot \cos(2\omega t + \varphi)$

The mean value of the instantaneous power p is the real power P dissipated by the circuit resistors, and equal to the formula:

$P = V \cdot I \cdot \cos\varphi$ (W), $\cos\varphi =$ power factor

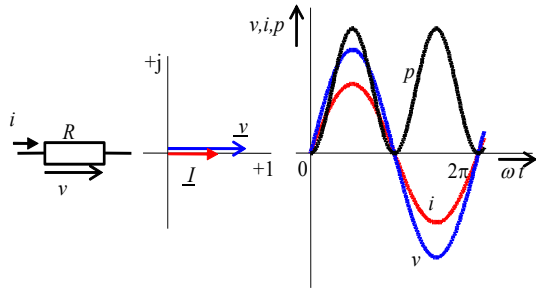
The power calculated using $\sin\varphi$ is called reactive power Q, it is associated with the circuit reactance and represents an exchange of energy between the source and the circuit reactance, so its mean value is equal to zero. The total power in the circuit is called apparent power S (G. Rizzoni: Fundamentals of electrical engineering).

$$Q = V \cdot I \cdot \sin\varphi\,(\text{var}),\quad S = V \cdot I\ \ (\text{V}\cdot\text{A}),\quad S = \sqrt{P^2 + Q^2}$$

2.4 Methods of solving AC electrical circuits

To solve the linear AC electrical circuits a behavior of the ideal resistor, inductor and capacitor in a circuit is necessary to demonstrate (G. Rizzoni: Fundamentals of electrical engineering).

2.4.1 The ideal resistor



In the picture the circuit symbol, phasor diagram and current, voltage and power waveforms are shown.

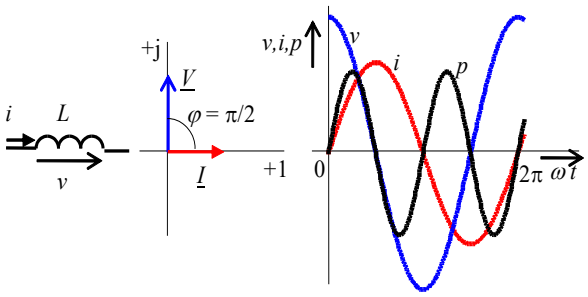
$$v = R \cdot i = R \cdot I_{\text{m}} \cdot \sin\omega t = V_{\text{m}} \cdot \sin\omega t$$

$$V_{\text{m}} = R \cdot I_{\text{m}}$$

$$R = \frac{v}{i} = \frac{V}{I} \rightarrow V = R \cdot I$$

$$\varphi = 0 \rightarrow \cos\varphi = 1 \rightarrow P = V \cdot I = R \cdot I^2$$

2.4.2 The ideal inductor



In the picture the circuit symbol, phasor diagram and current, voltage and power waveforms are shown.

$$v = L \cdot \frac{\text{d}\,i}{\text{d}\,t} = V_{\text{m}} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

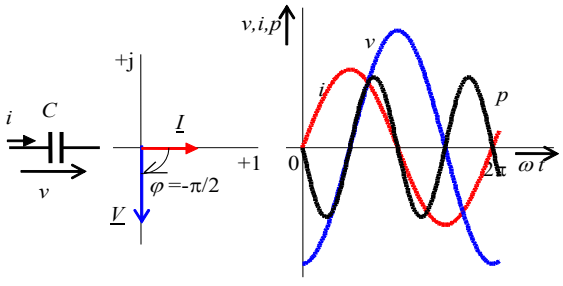
$$V_{\text{m}} = \omega \cdot L \cdot I_{\text{m}} = X_{\text{L}} \cdot I_{\text{m}} \rightarrow\ X_{\text{L}} = \omega \cdot L\,(\Omega)$$

$$V = X_{\text{L}} \cdot I$$

$$\varphi = 90^0 \rightarrow \cos\varphi = 0 \rightarrow P = 0$$

$$\sin\varphi = 1 \rightarrow\ Q = V \cdot I = X_{\text{L}} \cdot I^2$$

2.4.3 The ideal capacitor



In the picture the circuit symbol, phasor diagram and current, voltage and power waveforms are shown.

$$i = C \cdot \frac{\text{d}\,v}{\text{d}\,t} \rightarrow v = \frac{1}{C}\int i \cdot \text{d}\,t = V_{\text{m}} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{\text{m}} = \frac{1}{\omega \cdot C} \cdot I_{\text{m}} = X_{\text{C}} \cdot I_{\text{m}} \rightarrow X_{\text{C}} = \frac{1}{\omega \cdot C}\ \ (\Omega),\ V = X_{\text{C}} \cdot I$$

$$\varphi = 90^0 \rightarrow \cos\varphi = 0 \rightarrow P = 0$$

$$\sin\varphi = 1 \rightarrow Q = V \cdot I = X_{\text{C}} \cdot I^2$$

In a simple electrical circuit with elements connected only in series or in parallel the total resistance (=impedance), the total voltage or current can be solved by using the Pythagorean Theorem. For more complex circuits the phasor solution using complex numbers for defining parameters of circuit elements (impedances, voltages and currents) is the effective tool. For element impedances it is: $\underline{R} = R, \underline{X_{\text{L}}} = \text{j}X_{\text{L}}, \underline{X_{\text{C}}} = -\text{j}X_{\text{C}}$ (G. Rizzoni: Fundamentals of electrical engineering).